

Quantum noise and large-scale cosmic microwave background anisotropy

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Abstract

We propose a new source for the cosmological density perturbation which is passive fluctuations of the inflaton driven dynamically by a colored quantum noise as a result of its coupling to other massive quantum fields. The created fluctuations grow with time during inflation before horizon-crossing. However, the larger-scale modes cross out the horizon earlier, thus resulting in a suppression of their density perturbation as compared with those on small scales. By using current observed CMB data to constrain the parameters introduced, we find that a significant contribution from the noise-driven perturbation to the density perturbation is still allowed. It in turn gives rise to a suppression of the large-scale CMB anisotropy that may be relevant to the observed low quadrupole in the WMAP CMB anisotropy data. We also briefly discuss the implications to the energy scale of inflation and the spectral index and non-Gaussianity of the density perturbation.

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I. INTRODUCTION

The inflationary scenario [1], in which the present Universe is only a small local patch of a causally connected region at early time which underwent an exponential expansion driven by the inflaton potential, is generally accepted for explaining the observed spatially flat and homogeneous Universe. In addition, its quantum fluctuations during inflation give rise to primordial Gaussian density fluctuations with a nearly scale-invariant power spectrum, which is consistent with recent astrophysical and cosmological observations such as structure formation and CMB anisotropies [2].

Despite its success, some basic problems still remain unsolved. What is the origin of the inflaton potential? Do classical density inhomogeneities that we observe today really come from quantum fluctuations of the inflaton? To answer these questions is a big challenge to both theories and observations. The CMB anisotropy measurements made by COBE [3] and recently confirmed by WMAP [4] have shown that the amplitudes of the low- l multipoles are lower than expected for the Λ CDM model, although the statistical significance of such an anomaly is not large [5]. If this is true, it will give a clue to probe the physics of inflation.

There are many proposed solutions for the anomaly, mostly based on some *ad hoc* new ingredients in the generating process of density perturbation [6]. Recently the colored noise has been considered to explain the anomaly in the context of stochastic inflation [7]. In the stochastic approach to inflation [9], the inflaton field is coarse-grained by a Heaviside window function with a sharp frequency cutoff. Thus, the high-frequency modes constitute a white noise which drives the dynamics of the coarse-grained inflaton and generates a scale-invariant density power spectrum. In Ref. [7], they have adopted instead a Gaussian window function with a width characterizing the size of the coarse-grained domain [10] which is then arbitrarily chosen to be comparable to the Hubble radius. This smooth window function results in a colored noise which generates fewer fluctuations than the white noise on the scales slightly less than the chosen coarse-grained domain. So, they find a blue tilt of the power spectrum on large scales which can be tuned to fit the WMAP large-scale anisotropy data. Nevertheless, in this approach they inevitably resort to an *ad hoc* smoothing window function and therefore the width of the window function remains undetermined.

Apart from this free-field treatment, however, the inflaton is expected to be interacting with other fields; otherwise, the potential energy stored in the inflaton field cannot be converted into radiation after inflation. Cosmological phenomena associated with an interacting inflaton have been immensely studied in the context of reheating, preheating, backreactions to the inflaton dynamics, and etc. [11]. In particular, the influence from the interaction with other fields on density fluctuations during inflation has been discussed [12]. These have not only provided us with a field theoretical framework to understand the underlying physics of the inflation, but also explored the possibility of generating primordial density fluctuations with statistical properties deviated from both the scale-invariance and the Gaussianity.

In this paper, we will study the possible effects of coupling massive quantum fields to inflation and constrain the couplings and mass parameters by using observed CMB data. We will show that the inflaton fluctuations driven by the colored noise due to the interaction with the other field are strongly dependent on the onset of inflation and become scale-invariant asymptotically at small scales. This is due to the fact that the earlier a Fourier mode of the noise leaves the horizon the shorter the time can it drive the growth of the corresponding Fourier mode of the inflaton fluctuations. However, these passive fluctuations would not grow arbitrarily with time but only get boosted in an intrinsic time scale. This would result

in a suppression of the density power spectrum on large scales which may give a theoretical support to the low CMB low- l multipoles, if the passive ones contribute a significant portion to the density perturbation.

II. INFLUENCE FUNCTIONAL APPROACH

We will adopt the influence functional method [13] to take into account the quantum fluctuation effects from the massive fields coupled to the inflaton in a real-time manner, instead of using an one-loop effective potential as usually found for example in Ref. [14]. Thus, the effective Langevin equation of the inflaton is obtained, describing the time dependent corrections of the slow-rolling dynamics of the inflaton originally given by the classical inflaton potential. In particular, this Langevin equation, which goes beyond the mean field approximation, involves a stochastic noise term that may drive the growth of perturbation of the inflaton as we will see later.

A. Our model and Langevin equation for inflaton

Let us consider a slow-rolling inflaton ϕ coupled to a massive scalar field σ with a Lagrangian given by

$$\mathcal{L} = \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + \frac{1}{2}g^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma - V(\phi) - \frac{m_\sigma^2}{2}\sigma^2 - \frac{g^2}{2}\phi^2\sigma^2, \quad (1)$$

where $V(\phi)$ is the inflaton potential that complies with the slow-roll conditions and g is a coupling constant. Thus, we can approximate the space-time during inflation by a de Sitter metric given by

$$ds^2 = a^2(\eta)(d\eta^2 - d\mathbf{x}^2), \quad (2)$$

where η is the conformal time and $a(\eta) = -1/(H\eta)$ with H being the Hubble parameter. Here we rescale $a = 1$ at the initial time of the inflation era, $\eta_i = -1/H$. As for the system-environment field splitting, one may separate the inflaton field into its low- and high-frequency modes by introducing a window function. The low-frequency modes of cosmological relevance can be regarded as the system of interest whereas the high-frequency inflaton modes as well as the field σ are treated as an environment being traced out to the extent in how they influence the system. Nevertheless, the perturbation of the inflaton of cosmological relevant scales are found to be insignificantly affected from its high-frequency counterparts due to inflaton self-interaction [12]. Thus, here we restrict ourselves to the influence of quantum fluctuations from the field σ on the inflaton perturbation. In addition, the coupling of the inflaton to σ results in renormalization of the σ mass term due to quantum fluctuations of the inflaton, which will be included in the later discussion.

To proceed, let us assume that the initial density matrix at time η_i can be factorized as

$$\rho(\eta_i) = \rho_\phi(\eta_i) \otimes \rho_\sigma(\eta_i). \quad (3)$$

The full density matrix evolves unitarily and the evolution can be described by employing the closed-time-path formalism. Following the influence functional approach [12, 13], we

trace out the field σ in the perturbative expansion. The reduced density matrix of the system then becomes

$$\rho_r(\phi_f, \phi'_f; \eta_f) = \int d\phi_i d\phi'_i \mathcal{J}(\phi_f, \phi'_f, \eta_f; \phi_i, \phi'_i, \eta_i) \rho_r(\phi_i, \phi'_i; \eta_i), \quad (4)$$

where the propagating function $\mathcal{J}(\phi_f, \phi'_f, \eta_f; \phi_i, \phi'_i, \eta_i)$ is obtained as

$$\mathcal{J}(\phi_f, \phi'_f, \eta_f; \phi_i, \phi'_i, \eta_i) = \int_{\phi_i}^{\phi_f} \mathcal{D}\phi^+ \int_{\phi'_i}^{\phi'_f} \mathcal{D}\phi^- e^{i(S_0[\phi^+] - S_0[\phi^-])} \times e^{iS_{IF}[\phi^+, \phi^-]} \quad (5)$$

and the action for the field ϕ is given by

$$S_0[\phi] = \int d^4x a^2(\eta) \left[\frac{1}{2} \left(\frac{d\phi}{d\eta} \right)^2 - \frac{1}{2} (\nabla\phi)^2 - a^2(\eta) V(\phi) \right]. \quad (6)$$

Here we obtain the influence functional up to order g^4 as

$$\begin{aligned} e^{iS_{IF}[\phi^+, \phi^-]} = & \exp \left\{ -i \frac{g^2}{2} \int d^4x_1 a^4(\eta_1) \left[\phi^{+2}(x_1) \langle \sigma^+(x_1) \sigma^+(x_1) \rangle - \phi^{-2}(x_1) \langle \sigma^-(x_1) \sigma^-(x_1) \rangle \right] \right. \\ & - \frac{g^4}{4} \int d^4x_1 \int d^4x_2 a^4(\eta_1) a^4(\eta_2) \\ & \left[\phi^{+2}(x_1) \langle \sigma^+(x_1) \sigma^+(x_2) \rangle^2 \phi^{+2}(x_2) - \phi^{+2}(x_1) \langle \sigma^+(x_1) \sigma^-(x_2) \rangle^2 \phi^{-2}(x_2) \right. \\ & \left. \left. - \phi^{-2}(x_1) \langle \sigma^-(x_1) \sigma^+(x_2) \rangle^2 \phi^{+2}(x_2) + \phi^{-2}(x_1) \langle \sigma^-(x_1) \sigma^-(x_2) \rangle^2 \phi^{-2}(x_2) \right] \right\}. \quad (7) \end{aligned}$$

The Green's functions of the σ field are defined by

$$\begin{aligned} \langle \sigma^+(x) \sigma^+(x') \rangle &= \langle \sigma(x) \sigma(x') \rangle \theta(\eta - \eta') + \langle \sigma(x') \sigma(x) \rangle \theta(\eta' - \eta), \\ \langle \sigma^-(x) \sigma^-(x') \rangle &= \langle \sigma(x') \sigma(x) \rangle \theta(\eta - \eta') + \langle \sigma(x) \sigma(x') \rangle \theta(\eta' - \eta), \\ \langle \sigma^+(x) \sigma^-(x') \rangle &= \langle \sigma(x) \sigma(x') \rangle, \\ \langle \sigma^-(x) \sigma^+(x') \rangle &= \langle \sigma(x') \sigma(x) \rangle, \end{aligned} \quad (8)$$

and can be explicitly constructed as long as its vacuum state has been specified. To obtain the semiclassical Langevin equation, it is more convenient to introduce the average and relative field variables:

$$\phi = \frac{1}{2}(\phi^+ + \phi^-), \quad \phi_\Delta = \phi^+ - \phi^-. \quad (9)$$

The coarse-grained effective action (CGEA) including the influence action S_{IF} obtained from Eqs. (5) and (7) is then given by

$$\begin{aligned} S_{CGEA}[\phi, \phi_\Delta] = & \int d^4x a^2(\eta) \phi_\Delta(x) \left\{ -\ddot{\phi}(x) - 2aH\dot{\phi}(x) + \nabla^2\phi(x) - a^2 \left[V'(\phi) + g^2 \langle \sigma^2 \rangle \phi(x) \right] \right. \\ & \left. - g^4 a^2(\eta) \phi(x) \int d^4x' a^4(\eta') \theta(\eta - \eta') iG_-(x, x') \phi^2(x') \right\} \\ & + i \frac{g^4}{2} \int d^4x \int d^4x' a^4(\eta) a^4(\eta') \phi_\Delta(x) \phi(x) G_+(x, x') \phi_\Delta(x') \phi(x') + \mathcal{O}(\phi_\Delta^3), \end{aligned} \quad (10)$$

where the dot and prime denote respectively differentiation with respect to η and ϕ . In addition, we have used the fact that correlation functions of the fields evaluated at the same space-time point in the $+$ and $-$ branches are equal, namely, $\langle \sigma^+(x_1) \sigma^+(x_1) \rangle = \langle \sigma^-(x_1) \sigma^-(x_1) \rangle \equiv \langle \sigma^2(x_1) \rangle$. The kernels G_\pm can be obtained from the Green's function of σ :

$$G_\pm(x, x') = \langle \sigma(x) \sigma(x') \rangle^2 \pm \langle \sigma(x') \sigma(x) \rangle^2. \quad (11)$$

The imaginary part of the above influence action can be re-expressed by introducing an auxiliary field ξ with a distribution function of the Gaussian form,

$$P[\xi] = \exp \left\{ -\frac{1}{2} \int d^4x \int d^4x' \xi(x) G_+^{-1}(x, x') \xi(x') \right\}, \quad (12)$$

leading to

$$e^{iS_{CGEA}} = \int \mathcal{D}\xi P[\xi] \exp iS_{\text{eff}}[\phi, \phi_\Delta, \xi], \quad (13)$$

with the effective action S_{eff} given by

$$\begin{aligned} S_{\text{eff}}[\phi, \phi_\Delta, \xi] &= \int d^4x a^2(\eta) \phi_\Delta(x) \left\{ -\ddot{\phi}(x) - 2aH\dot{\phi}(x) + \nabla^2 \phi(x) - a^2 [V'(\phi) + g^2 \langle \sigma^2 \rangle \phi(x)] \right. \\ &\quad \left. - g^4 a^2(\eta) \phi(x) \int d^4x' a^4(\eta') \theta(\eta - \eta') iG_-(x, x') \phi^2(x') + g^2 a^2(\eta) \phi(x) \xi(x) \right\} \end{aligned} \quad (14)$$

The semiclassical approximation requires to extremize the effective action $\delta S_{\text{eff}}/\delta \phi_\Delta$ when long-wavelength inflaton modes of cosmological interest have gone through the quantum-to-classical transition due to the rapid expansion of the scale factor [8]. Then, we obtain the semiclassical Langevin equation for ϕ :

$$\begin{aligned} \ddot{\phi} + 2aH\dot{\phi} - \nabla^2 \phi + a^2 [V'(\phi) + g^2 \langle \sigma^2 \rangle \phi] - g^4 a^2 \phi \int d^4x' a^4(\eta') \times \\ \theta(\eta - \eta') iG_-(x, x') \phi^2(x') = g^2 a^2 \phi \xi + \xi_w, \end{aligned} \quad (15)$$

where we have included the white noise in the free-field stochastic inflation [9]. The white noise ξ_w reproduces the active or intrinsic inflaton quantum fluctuations $\langle \varphi_q^2 \rangle$ with a scale-invariant power spectrum given by $\Delta_k^q = H^2/(4\pi^2)$ [15]. The effects from the quantum field σ on the inflaton are given by the dissipation via the kernel G_- as well as a stochastic force induced by the multiplicative colored noise ξ with

$$\langle \xi(x) \xi(x') \rangle = G_+(x, x'). \quad (16)$$

Note that the white noise is uncorrelated with the colored noise since $\langle \xi_w(x) \xi(x') \rangle \sim \langle \phi(x) \sigma^2(x') \rangle = 0$.

B. Approximate solutions

To solve Eq. (15), let us first drop the dissipative term which we will discuss later and consider the colored noise only. Then, after decomposing ϕ into a mean field and a classical perturbation: $\phi(\eta, \mathbf{x}) = \bar{\phi}(\eta) + \varphi(\eta, \mathbf{x})$, we obtain the linearized Langevin equation,

$$\ddot{\varphi} + 2aH\dot{\varphi} - \nabla^2 \varphi + a^2 m_{\varphi\text{eff}}^2 \varphi = g^2 a^2 \bar{\phi} \xi, \quad (17)$$

where the effective mass is $m_{\varphi_{\text{eff}}}^2 = V''(\bar{\phi}) + g^2\langle\sigma^2\rangle$ and the time evolution of $\bar{\phi}$ is governed by $V(\bar{\phi})$. The equation of motion for σ from which we construct its Green's function can be read off from its quadratic terms in the Lagrangian (1) as

$$\ddot{\sigma} + 2aH\dot{\sigma} - \nabla^2\sigma + a^2m_\sigma^2\sigma = 0. \quad (18)$$

Let us decompose

$$\begin{aligned} Y(x) &= \int \frac{d^3\mathbf{k}}{(2\pi)^{\frac{3}{2}}} Y_{\mathbf{k}}(\eta) e^{i\mathbf{k}\cdot\mathbf{x}}, \quad \text{where } Y = \varphi, \xi, \\ \sigma(x) &= \int \frac{d^3\mathbf{k}}{(2\pi)^{\frac{3}{2}}} [b_{\mathbf{k}}\sigma_k(\eta) e^{i\mathbf{k}\cdot\mathbf{x}} + \text{h.c.}], \end{aligned} \quad (19)$$

where $b_{\mathbf{k}}^\dagger$ and $b_{\mathbf{k}}$ are creation and annihilation operators satisfying $[b_{\mathbf{k}}, b_{\mathbf{k}'}^\dagger] = \delta(\mathbf{k} - \mathbf{k}')$. Then, the solution to Eq. (17) is obtained as

$$\varphi_{\mathbf{k}}(\eta) = -ig^2 \int_{\eta_i}^{\eta} d\eta' a^4(\eta') \bar{\phi}(\eta') \xi_{\mathbf{k}}(\eta') [\varphi_k^1(\eta') \varphi_k^2(\eta) - \varphi_k^2(\eta') \varphi_k^1(\eta)], \quad (20)$$

where the homogeneous solutions $\varphi_k^{1,2}$ are given by

$$\varphi_k^{1,2} = \frac{1}{2a} (\pi|\eta|)^{\frac{1}{2}} H_\nu^{(1),(2)}(k\eta). \quad (21)$$

Here $H_\nu^{(1)}$ and $H_\nu^{(2)}$ are Hankel functions of the first and second kinds respectively and $\nu^2 = 9/4 - m_{\varphi_{\text{eff}}}^2/H^2$. In addition, we have from Eq. (18) that

$$\sigma_k(\eta) = \frac{1}{2a} (\pi|\eta|)^{\frac{1}{2}} [c_1 H_\mu^{(1)}(k\eta) + c_2 H_\mu^{(2)}(k\eta)], \quad (22)$$

where the constants c_1 and c_2 are subject to the normalization condition, $|c_2|^2 - |c_1|^2 = 1$, and $\mu^2 = 9/4 - m_\sigma^2/H^2$.

III. POWER SPECTRUM OF PASSIVE INFLATON FLUCTUATIONS

Now we are able to calculate the power spectrum of the perturbation φ . To maintain the slow-roll condition: $m_{\phi_{\text{eff}}}^2 = m_{\varphi_{\text{eff}}}^2 \ll H^2$ (i.e., $\nu = 3/2$), we require that $g^2 < 1$ and $m_\sigma^2 > H^2$. The latter condition limits the growth of $\langle\sigma^2\rangle$ during inflation to be less than about $10^{-2}H^2$ [16, 17]. This amounts to a small contribution to the slow-roll parameter $\eta \equiv M_{Pl}^2 V''/V$, where M_{Pl} is the reduced Planck mass, of order $10^{-2}g^2/3 < 0.3\%$ consistent with the WMAP measurements of the density power spectral index that determine the slow-roll parameters only up to a few percent level [4].

In Eq. (18), we have not considered mass corrections to m_σ^2 from the mean inflaton field, $g^2\bar{\phi}^2$, and the mass renormalization due to quantum fluctuations of the inflaton, $g^2\langle\varphi_q^2\rangle$. Under the slow-roll condition, $\langle\varphi_q^2\rangle$ grows linearly as $H^3 t/4\pi^2$ [16, 17] and thus $\langle\varphi_q^2\rangle \simeq H^2$ after about 60 efoldings (i.e., $Ht \simeq 60$). Therefore, as long as $g^2\bar{\phi}^2 \leq 2H^2$ for the period during which those k modes of cosmologically relevant scales cross out the horizon, we can conveniently choose $m_\sigma^2 = 2H^2$ (i.e., $\mu = 1/2$) for which σ takes a very simple form. After

then, $g^2\bar{\phi}^2$ may grow to a value much bigger than H^2 and thus the effective mass of σ becomes much larger than H^2 . If so, this large mass will suppress the growth of $\langle\sigma^2\rangle$ [17] and may diminish the effect of the noise term. From now on, let us consider only the relevant period with $g^2\bar{\phi}^2 \leq 2H^2$. It was shown that when $\mu = 1/2$ one can select the Bunch-Davies vacuum (i.e., $c_2 = 1$ and $c_1 = 0$) [17]. Hence, using Eqs. (16) and (20), we obtain

$$\langle\varphi_{\mathbf{k}}(\eta)\varphi_{\mathbf{k}'}^*(\eta)\rangle = \frac{2\pi^2}{k^3}\Delta_k^\xi(\eta)\delta(\mathbf{k}-\mathbf{k}'), \quad (23)$$

where the noise-driven power spectrum is given by

$$\Delta_k^\xi(\eta) = \frac{g^4 z_-^2}{8\pi^4} \int_{z_i}^z dz_1 \int_{z_i}^z dz_2 \bar{\phi}(\eta_1)\bar{\phi}(\eta_2) \frac{\sin z_-}{z_1 z_2 z_-} [\sin(2\Lambda z_-/k)/z_- - 1] F(z_1)F(z_2), \quad (24)$$

where $z_- = z_2 - z_1$, $z = k\eta$, $z_i = k\eta_i = -k/H$, Λ is the momentum cutoff introduced in the evaluation of the ultraviolet divergent k -integration of σ_k in the Green's function (11), and

$$F(y) = \left(1 + \frac{1}{yz}\right) \sin(y-z) + \left(\frac{1}{y} - \frac{1}{z}\right) \cos(y-z). \quad (25)$$

Note that the term $\sin(2\Lambda z_-/k)/z_- \simeq \pi\delta(z_-)$ when $\Lambda \gg k$, so $\Delta_k^\xi(\eta)$ is insensitive to Λ . Both $\bar{\phi}(\eta_1)$ and $\bar{\phi}(\eta_2)$ in Eq. (24) can be approximated as a constant mean field $\bar{\phi}_0$. It is because we are concerned with large scales at which the rate of change of the mean field at horizon-crossing is given by $d\bar{\phi}/d\ln k \simeq -\sqrt{2\epsilon}M_{Pl}$, where the slow-roll parameter $2\epsilon \equiv M_{Pl}^2(V'/V)^2$. The low value of ϵ is crucial in determining the observed density power spectrum consistent with current measurements [18]. In fact, it is found to be consistent with zero up to the scale near the first CMB Doppler peak in the WMAP measurements [4]. Then, we plot $\Delta_k^\xi(\eta)$ at the horizon-crossing time given by $z = -2\pi$ versus k/H in Fig. 1. The figure shows that the noise-driven fluctuations depend on the onset time of inflation and approach asymptotically to a scale-invariant power spectrum $\Delta_k^\xi \simeq 0.2g^4\bar{\phi}_0^2/(4\pi^2)$ at large k .

At this point, let us examine the dissipation term in the Langevin equation (15), which is actually divergent. We have removed the divergency by using the regularization method [17, 19] that sets the ultraviolet cutoff $\Lambda = He^{Ht}$, which includes all the modes with wavelength greater than the horizon at time t during inflation, as these are the ones responsible for the growth of $\langle\sigma^2\rangle$ (Note that we have used the same method to regularize the divergent $\langle\sigma^2\rangle$ above). Hence, we have found that this term only contributes a mass correction of about $10^{-2}g^4\bar{\phi}_0^2$ to $m_{\varphi\text{eff}}^2$ as well as a small friction term of order $10^{-2}g^4\bar{\phi}_0^2 a\dot{\phi}/H$ to Eq. (15). This term also gives a correction to the slope of the inflaton potential $V'(\phi)$ of order $10^{-2}g^4\bar{\phi}_0^3$, which in turn changes the slow-roll parameter ϵ by $10^{-4}g^8\bar{\phi}_0^6/(2H^4M_{Pl}^2)$. All of these corrections can be neglected as long as $g^2\bar{\phi}_0^2 \leq 2H^2$.

IV. EFFECTS ON LARGE-SCALE CMB ANISOTROPY

Our results show novel effects on the large-scale density perturbation. Within the present model, the value of $g^4\bar{\phi}_0^2$ is not fixed except that $g^2\bar{\phi}_0^2 \leq 2H^2$, which is to keep the condition, $m_\sigma^2 = 2H^2$, as well as to make sure that the time-dependent corrections to both the mass scales and the slow-roll parameters are small. If $g^2 \lesssim 1$ and $g^4\bar{\phi}_0^2 \simeq 2H^2$, then Δ_k^ξ can be as

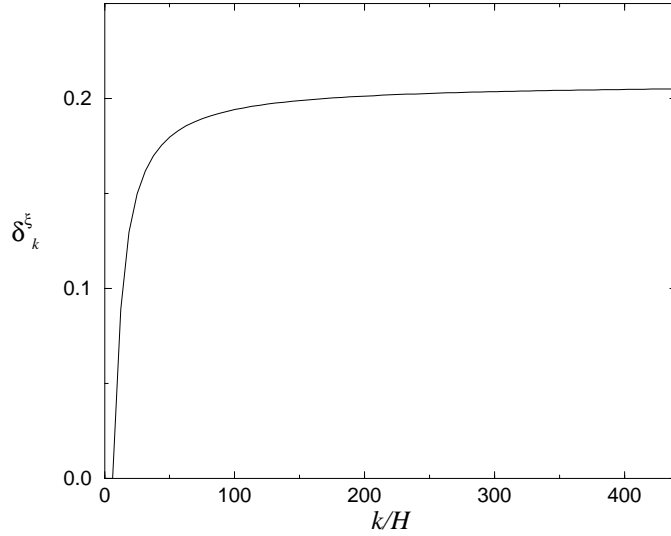


FIG. 1: Power spectrum of the noise-driven inflaton fluctuations $\delta_k^\xi \equiv 4\pi^2 \Delta_k^\xi / g^4 \bar{\phi}_0^2$. The starting point, $k/H = 2\pi$, corresponds to the k -mode that leaves the horizon at the start of inflation.

large as $0.4\Delta_k^q$ at large k . Notice that the perturbative expansion parameter is $g^2/(2\pi) \ll 1$ such that higher-order contributions in g can be safely ignored. In the standard slow-roll inflation, the density power spectrum induced by the active quantum fluctuations is nearly scale-invariant and is given by $P_k^q = X(\bar{\phi})\Delta_k^q$, where $X(\bar{\phi})$ is determined by the slow-roll kinematics [1], whereas the noise-driven density power spectrum, $P_k^\xi = X(\bar{\phi})\Delta_k^\xi$, is blue-tilted on large scales. Assuming a total density power spectrum $P_k = P_k^q + P_k^\xi$ with $X(\bar{\phi} = \bar{\phi}_0)$ set to constant and using the set of cosmological parameters measured by WMAP [4], we have run the CMBFAST numerical codes [20] to compute the CMB anisotropy power spectrum as shown in Fig. 2, where we have set $g^4 \bar{\phi}_0^2 = 2H^2$. We can see that a scale-invariant power spectrum at small scales can be achieved by properly choosing the initial time of inflation. For the dashed (dotted) curve, the Fourier mode $k/H = 500\pi$ ($k/H = 50\pi$) corresponding to the physical scale of 0.05Mpc^{-1} crosses out the horizon at about 5.5 (3.2) efoldings. Meanwhile, the suppressed large-scale density perturbation can account for the low CMB low- l multipoles.

V. IMPLICATIONS

Let us briefly mention other implications to the inflationary cosmology which deserve investigations to further constrain the free parameters here. We rewrite $P_k = P_k^q(1 + \Delta_k^\xi/\Delta_k^q)$. Therefore, the energy scale of inflation inferred from measurements of the density power spectrum will be lower than the standard slow-roll prediction by a factor $(1 + \Delta_k^\xi/\Delta_k^q)^{-1/4} \simeq 0.92$. The spectral index is then given by

$$n(k) - 1 \equiv \frac{d \ln P_k}{d \ln k} = \frac{d \ln P_k^q}{d \ln k} + \frac{d}{d \ln k} \ln \left(1 + \frac{\Delta_k^\xi}{\Delta_k^q} \right). \quad (26)$$

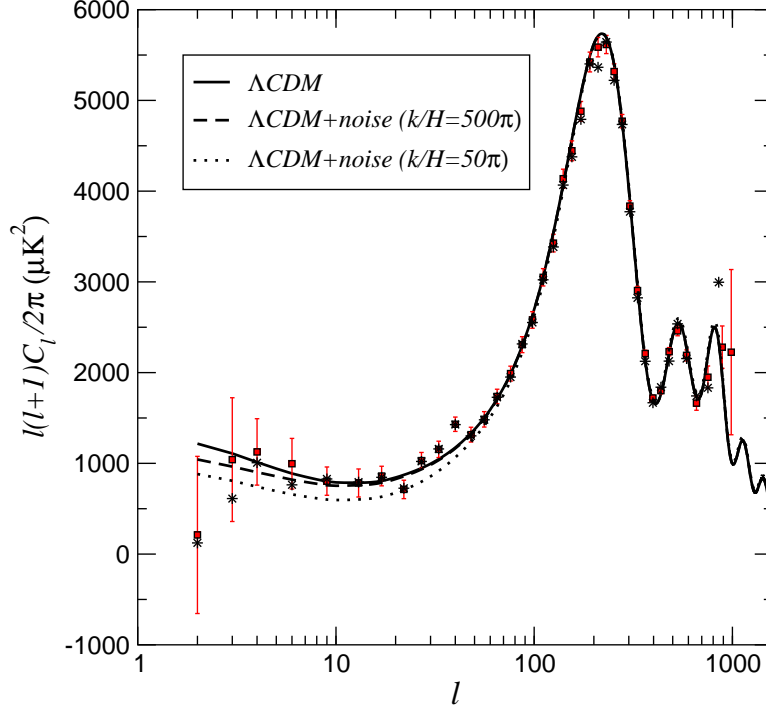


FIG. 2: CMB anisotropy in the Λ CDM model with the density power spectrum $P_k = P_k^q + P_k^\xi$. The solid curve is the Λ CDM model with a scale-invariant P_k^q induced by quantum fluctuations. The dashed and dotted curves represent respectively noise-driven P_k^ξ 's with $k/H = 500\pi$ and $k/H = 50\pi$ corresponding to 0.05Mpc^{-1} . We normalize all the anisotropy spectra at the first Doppler peak. Also shown are the three-year WMAP data including error bars and the first-year WMAP data denoted by stars [4].

The standard slow-roll inflation predicts that $|n(k) - 1| \gg |d \ln n(k)/d \ln k|$, although the three-year WMAP results have indicated that $|n(k) - 1| \simeq |d \ln n(k)/d \ln k|$ [4]. There have been several approaches to reduce the discrepancy by either violating or generalizing the slow-roll condition for P_k^q [21]. Unlike these kinematic approaches, the additional term containing Δ_k^ξ in Eq. (26) offers a new dynamical source for breaking the scale invariance. It is worthwhile to study the overall effect of the corrections induced by the dissipation, the static mass approximation, and the static mean field approximation to the spectral index. Especially, we expect a running spectral index at large k due to dissipational effects on the fluctuations at late times. Furthermore, it is interesting to consider the three-point function $\langle \varphi_{\mathbf{k}_1} \varphi_{\mathbf{k}_2} \varphi_{\mathbf{k}_3} \rangle$ by invoking higher-order terms in the presence of inflaton mean field.

VI. CONCLUSIONS AND DISCUSSIONS

We have discussed the effect of an interacting inflaton to the cosmological density perturbation. The passive inflaton fluctuations induced by the interaction are found to be blue-tilted on large-scales. This results in a suppression of the large-scale CMB anisotropy that may be relevant to the observed low quadrupole in the WMAP CMB anisotropy data. Interestingly, the observed low CMB quadrupole may open a window on the physics of the first few efoldings of inflation. The results are obtained by solving the Langevin equa-

tion (15) of the inflaton arising from its coupling to a massive quantum field with mass of the order of the Hubble scale. Here we have not specified the inflaton potential. However, we restrict ourselves to the situation typically for a small-field inflation, in which the mean field $\bar{\phi}^2 \simeq H^2$ when inflaton modes of cosmologically interesting scales cross out the horizon. Presently, observational data prefer a slow-roll inflaton potential; however, the exact form of the potential is still elusive. We expect that the above mentioned situation can be realized in certain inflationary models. Moreover, our assumption on the values of the introduced parameters allows us to obtain an analytic solution of the Langevin equation. So, it would be interesting to relax this assumption by performing a full analysis of the cosmological effects due to the interacting quantum environment in a viable inflation model.

In fact, one can also consider the effect of a coupling scalar field to the inflaton in the context of large-field inflation such as chaotic inflation [18]. We have shown that the corrections from the coupling scalar field to the slow-roll parameters are small for $g^2 \bar{\phi}_0^2 \leq 2H^2$. Hence, the coupling constant g can be small for those large values of ϕ_0 of order of M_{Pl} as usually found in chaotic inflation; however, this will give an insignificant correction to the density power spectrum. For a strong coupling of $g^2 \lesssim 1$ as found for example in the hybrid inflation [22], the field σ obtains a huge effective mass from the inflaton mean field much greater than H . This case goes beyond our assumption and needs further study. Finally, although we have worked with a simple inflaton-scalar interaction, our results should be generic to any interacting model.

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